

**EJEMPLO 3:**

Calcular  $e^0 = e^{\begin{pmatrix} 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{pmatrix}}$ .

**RESOLUCIÓN:**

$$e^0 = I + 0 + \frac{1}{2!}(0)^2 + \frac{1}{3!}(0)^3 + \dots = I$$

Adoptando  $A^0 = I$ , la anterior permite escribir  $e^{A_s} = \sum_{n=0}^{\infty} \frac{1}{n!} A_s^n$  y  $e^{A_s t} = \sum_{n=0}^{\infty} \frac{1}{n!} A_s^n t^n$ .



Bertossi, Pastorelli, Casco

**EJEMPLO 4:**

Siendo  $D = \begin{pmatrix} a_{11} & 0 & \cdots & 0 \\ 0 & a_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_{nn} \end{pmatrix}$  mostrar que  $e^{Dt} = \begin{pmatrix} e^{a_{11}t} & 0 & \cdots & 0 \\ 0 & e^{a_{22}t} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & e^{a_{nn}t} \end{pmatrix}$ .

**RESOLUCIÓN:**

En el caso que  $A$  sea una matriz diagonal  $D = \begin{pmatrix} a_{11} & 0 & \cdots & 0 \\ 0 & a_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_{nn} \end{pmatrix}$ , calcular  $e^{Dt}$  es un problema

sencillo, dado que:

$$D^n = \begin{pmatrix} a_{11} & 0 & \cdots & 0 \\ 0 & a_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_{nn} \end{pmatrix}^n = \begin{pmatrix} a_{11}^n & 0 & \cdots & 0 \\ 0 & a_{22}^n & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_{nn}^n \end{pmatrix}$$

Luego:

$$e^{Dt} = \sum_{n=0}^{\infty} \frac{1}{n!} (Dt)^n = \sum_{n=0}^{\infty} \frac{1}{n!} D^n t^n = \sum_{n=0}^{\infty} \frac{t^n}{n!} \begin{pmatrix} a_{11}^n & 0 & \cdots & 0 \\ 0 & a_{22}^n & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_{nn}^n \end{pmatrix}$$

$$e^{Dt} = \begin{pmatrix} \sum_{n=0}^{\infty} \frac{1}{n!} (a_{11}t)^n & 0 & \cdots & 0 \\ 0 & \sum_{n=0}^{\infty} \frac{1}{n!} (a_{22}t)^n & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sum_{n=0}^{\infty} \frac{1}{n!} (a_{nn}t)^n \end{pmatrix} = \begin{pmatrix} e^{a_{11}t} & 0 & \cdots & 0 \\ 0 & e^{a_{22}t} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & e^{a_{nn}t} \end{pmatrix}$$

